Technical Appendix of
"Exchange Rates and Monetary Policy in Emerging Market Economies "

Not to be Published

1 Equilibrium

In this appendix, we provide a detailed outline of how the model of the paper is constructed.

1.1 Households

The household's budget constraint is described in 2.2 of the text.

The household optimality conditions for labor, domestic bonds, and foreign bonds are:

$$W_t = \eta H_t^{\psi} P_t C_t^{\sigma} \tag{1.1}$$

$$\frac{1}{1+i_{t+1}} = \beta E_t \left(\frac{C_t^{\sigma} P_t}{C_{t+1}^{\sigma} P_{t+1}} \right) \tag{1.2}$$

$$\frac{1}{1+i_{t+1}^*} \left[1 - \frac{\psi_D P_t}{S_t} (D_{t+1} - \bar{D}) \right] = \beta E_t \left\{ \frac{C_t^{\sigma} P_t}{C_{t+1}^{\sigma} P_{t+1}} \frac{S_{t+1}}{S_t} \right\} \tag{1.3}$$

where the price index is defined as :

$$P_t = (a P_{Nt}^{1-\rho} + (1-a) P_{Mt}^{1-\rho})^{\frac{1}{1-\rho}} \tag{1.4}$$

The household's non-tradable goods and tradable goods demand are:

$$C_{Nt} = a \left(\frac{P_{Nt}}{P_t}\right)^{-\rho} C_t \tag{1.5}$$

$$C_{Mt} = (1-a) \left(\frac{P_{Mt}}{P_t}\right)^{-\rho} C_t \tag{1.6}$$

1.2 Production Firms

Non-tradable goods firms have production functions given by

$$Y_{Nt} = A_N K_{Nt}^{\alpha} H_{Nt}^{\Omega(1-\alpha)} (H_{Nt}^e)^{(1-\Omega)(1-\alpha)} \tag{1.7}$$

Cost minimisation leads to the following implicit demand for both types of labor, and capital:

$$W_t = MC_{Nt}(1-\alpha)\Omega\frac{Y_{Nt}}{H_{Nt}} \tag{1.8}$$

$$W^e_{Nt} = MC_{Nt}(1-\alpha)(1-\Omega)\frac{Y_{Nt}}{H^e_{Nt}} \tag{1.9}$$

$$R_{Nt} = MC_{Nt}\alpha\frac{Y_{Nt}}{K_{Nt}} \tag{1.10}$$

Export sector firms have production function:

$$Y_{Xt} = A_X K^{\gamma}_{Xt} H^{\Omega(1-\gamma)}_{Xt}(H^e_{Xt})^{(1-\Omega)(1-\gamma)} \tag{1.11}$$

And

Cost minimisation in the export sector leads to demand for labor and capital:

$$W_t = P_{Xt}(1-\gamma)\Omega\frac{Y_{Xt}}{H_{Xt}} \tag{1.12}$$

$$W^e_{Xt} = P_{Xt}(1-\gamma)(1-\Omega)\frac{Y_{Xt}}{H^e_{Xt}} \tag{1.13}$$

$$R_{Xt} = P_{Xt}\gamma\frac{Y_{Xt}}{K_{Xt}} \tag{1.14}$$

1.3 Unfinished Capital Goods firms

These firms invest (where one unit of investment costs P_t, since the investment composite is of the same form as the consumption good) and rent capital to produce new unfinished capital goods for sale to entrepreneurs. Capital in each sector therefore receives a rental payment from unfinished capital goods firm as well as from final goods firms. Capital accumulation in each sector may be described as:

$$K_{Nt+1} = \phi(\frac{I_{Nt}}{K_{Nt}})K_{Nt} + (1-\delta)K_{Nt} \tag{1.15}$$

$$K_{Xt+1} = \phi(\frac{I_{Xt}}{K_{Xt}})K_{Xt} + (1-\delta)K_{Xt} \tag{1.16}$$

where $\phi(\frac{I^j_t}{K^j_t}) = \frac{I^j_t}{K^j_t} - \frac{\psi_I}{2}\left(\frac{I^j_t}{K^j_t} - \delta\right)^2$, and $j = X, N$.

Unfinished capital goods firms then have the CRS production functions given by $\phi(\frac{I_{Nt}}{K_{Nt}})K_{Nt}$ and $\phi(\frac{I_{Xt}}{K_{Xt}})K_{Xt}$. If the price of an unfinished capital good in the non-traded sector is Q_{Nt}, then the firm's profit maximisation implies that

$$Q_{Nt}\phi'(\frac{I_{Nt}}{K_{Nt}}) = P_t \tag{1.17}$$

$$Q_{Nt}\phi(\frac{I_{Nt}}{K_{Nt}}) - Q_{Nt}\phi'(\frac{I_{Nt}}{K_{Nt}})\frac{I_{Nt}}{K_{Nt}} = R^G_{KNt} \tag{1.18}$$

where R^G_{KNt} is defined as the rental rate that entrepreneurs receive for renting their current capital to unfinished capital goods firms.

The unfinished capital goods firms in the export sector have analogous decisions.

1.4 Price Setting

Profit maximising firms in the non-traded goods sector lead to the condition for price setting:

$$\begin{aligned} P_{Nt} &= \frac{\lambda}{\lambda-1}MC_{Nt} - \frac{\psi_{P_N}}{\lambda-1}\frac{P_t}{Y_{Nt}}\frac{P_{Nt}}{P_{Nt-1}}\left(\frac{P_{Nt}}{P_{Nt-1}}-1\right) + \\ &\quad \frac{\psi_{P_N}}{\lambda-1}E_t\left[\Gamma_{t+1}\frac{P_{t+1}}{Y_{Nt}}\frac{P_{Nt+1}}{P_{Nt}}\left(\frac{P_{Nt+1}}{P_{Nt}}-1\right)\right] \end{aligned} \tag{1.19}$$

where Γ_t is the home nominal discount factor, defined by 2.20 in the text.

Assuming that importing goods firms face similar costs of price change, we get:

$$\begin{aligned} P_{Mt} &= \frac{\lambda}{\lambda-1}S_tP^*_{Mt} - \frac{\psi_{P_M}}{\lambda-1}\frac{P_t}{T_{Mt}}\frac{P_{Mt}}{P_{Mt-1}}\left(\frac{P_{Mt}}{P_{Mt-1}}-1\right) + \\ &\quad \frac{\psi_{P_M}}{\lambda-1}E_t\left[\Gamma_{t+1}\frac{P_{t+1}}{T_{Mt}}\frac{P_{Mt+1}}{P_{Mt}}\left(\frac{P_{Mt+1}}{P_{Mt}}-1\right)\right] \end{aligned} \tag{1.20}$$

where T_{Mt} is the demand for imports, $S_tP^*_{Mt}$ is the marginal cost for importers.

The export good price is determined on world markets as:

$$P_{Xt} = S_tP^*_{Xt} \tag{1.21}$$

1.5 The entrepreneur's problem:

The details of the optimal contract are derived in section 2 of the appendix below. Here we outline the specification of the entrepreneur's behavior that are important in the solution of the model.

The finance premium rp_{Nt+1} in the non-tradable sector (adjusted for exchange rate changes) is determined in the following equation:

$$E_t\left[R_{KNt+1}\frac{1}{rp_{Nt+1}}\right] = (1+i^*_{t+1}) \tag{1.22}$$

where

$$rp_{Nt+1} = \frac{E_t\left(\frac{A'(\bar{\omega_{Nt+1}})}{B'(\bar{\omega_{Nt+1}})}\frac{S_{t+1}}{S_t}\right)}{\left[B(\bar{\omega_{Nt+1}})\frac{A'(\bar{\omega_{Nt+1}})}{B'(\bar{\omega_{Nt+1}})} - A(\bar{\omega_{Nt+1}})\right]} \tag{1.23}$$

Here $A(\bar{\omega})$ is defined as the fraction of the return on capital that is obtained by entrepreneurial sector in the aggregate, and $B(\bar{\omega})$ is the fraction of the return that is obtained international lenders, net of the costs of monitoring. These functions are further defined below.

The participation constraint for international lenders is given by:

$$\frac{R_{KNt}S_{t-1}}{S_t}B(\bar{\omega_{Nt}}) = (1+i_t^*)(1-\frac{Z_{Nt}}{Q_{Nt-1}K_{Nt}}) \tag{1.24}$$

where Z_{Nt} is the net worth of entrepreneurs in the non-tradable sector.

Entrepreneurs die at rate $(1-\nu)$ and consume their return on capital if they die. The aggregate consumption of entrepreneurs in the non-tradable good sector is:

$$P_tC_t^{Ne} = (1-\nu)R_{KNt}Q_{Nt-1}K_{Nt}A(\bar{\omega_{Nt}}) \tag{1.25}$$

The evolution of net worth may be written as

$$\begin{aligned} Z_{Nt+1} &= \nu R_{KNt}Q_{Nt-1}K_{Nt}A(\bar{\omega_{Nt}}) + W_{Nt}^e \\ &= \nu R_{KNt}Q_{Nt-1}K_{Nt}\left(1-B(\bar{\omega_{Nt}})-\mu\int_0^{\bar{\omega_{Nt}}}\omega f(\omega)d\omega\right) + W_{Nt}^e \\ &= \nu(1-\phi_{Nt})R_{KNt}Q_{Nt-1}K_{Nt} - \nu(1+i_t^*)\frac{S_t}{S_{t-1}}(Q_{Nt-1}K_{Nt}-Z_{Nt}) + W_{Nt}^e \end{aligned} \tag{1.26}$$

where ϕ_{Nt} is the fraction of the payoff representing monitoring costs, and by 2.25 in the text $\frac{1}{S_{t-1}}(Q_{Nt-1}K_{Nt}-Z_{Nt}) = D_{Nt}^e$ represent foreign currency debt of the non-traded goods entrepreneurial sector. Then note that we may combine 1.25 and 1.26 to get the flow (aggregate) budget constraint of entrepreneurs in the non-traded goods sector as:

$$P_tC_t^{Ne} + Q_{Nt+1}K_{Nt+1} = S_tD_{Nt+1}^e + (1-\phi_{Nt})R_{KNt}Q_{Nt-1}K_{Nt} - (1+i_t^*)S_tD_{Nt}^e + W_{Nt}^e \tag{1.27}$$

which just says that total consumption, plus the purchase of capital goods, is equal to new foreign borrowing, plus the return on existing capital (net of monitoring costs) less the interest rate on existing foreign debt, plus wage income.

The rate of return for entrepreneurs in the non-traded sector consists of the rental return on capital received from the final goods sector as well as the unfinished capital goods sector, plus the value of undepreciated capital, divided by the original price of capital. This is

$$R_{KNt+1} = \frac{R_{Nt+1} + \left[1-\delta-\phi'(\frac{I_{Nt+1}}{K_{Nt+1}})\frac{I_{Nt+1}}{K_{Nt+1}} + \phi(\frac{I_{Nt+1}}{K_{Nt+1}})\right]Q_{Nt+1}}{Q_{Nt}} \tag{1.28}$$

1.6 Definition of $A(\bar{\omega})$, $B(\bar{\omega})$, and ϕ_{Nt}

$A(\cdot)$ is defined as the expected fraction of the return on capital accruing to the entrepreneur as part of the optimal contract. We may write is as:

$$A(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega f(\omega) d\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega$$

Likewise the return to the lender, net of monitoring costs, is

$$B(\cdot) = \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega + (1 - \mu) \int_{0}^{\bar{\omega}} \omega f(\omega) d\omega$$

We define ϕ_{Nt} as the fraction of the return on capital (in the non-tradeable sector) that is wasted in monitoring:

$$\phi_{Nt} = \mu \int_{0}^{\bar{\omega_{Nt}}} \omega f(\omega) d\omega$$

The case when ω_t^i is log-normally distributed with $E(ln\omega) = -\frac{\sigma_\omega^2}{2}$ and $Var(ln\omega) = \sigma_\omega^2$ is described in detail below.

The details of the entrepreneurial environment in the export sector are exactly analogous.

1.7 Interest Rate Rule

The monetary authority follows the interest rule given by:

$$1 + i_{t+1} = \left(\frac{P_{Nt}}{P_{Nt-1}} \frac{1}{\bar{\pi}_n} \right)^{\mu_{\pi n}} \left(\frac{P_t}{P_{t-1}} \frac{1}{\bar{\pi}} \right)^{\mu_\pi} \left(\frac{S_t}{\bar{S}} \right)^{\mu_S} (1 + \bar{i}) \tag{1.29}$$

1.8 Market Clearing and Balance of Payments

The non-tradable goods market clearing condition is written as total demand coming from consumers, firms, and entrepreneurs, including demand which is required to pay the costs of price adjustment (of both non-traded firms and foreign exporters), monitoring, and foreign bond adjustment[17]. Thus:

$$Y_{Nt} = a(\frac{P_{Nt}}{P_t})^{-\rho}[C_t + I_{Nt} + I_{Xt} + C_t^{Ne} + C_t^{Xe} + \frac{\psi_D}{2}(D_{t+1} - \bar{D})^2 + \frac{\psi_{P_N}}{2} \frac{(P_{Nt} - P_{Nt-1})^2}{P_{Nt-1}^2} + \frac{R_{KNt} Q_{Nt-1} K_{Nt}}{P_t} \phi_{Nt} + \frac{R_{KXt} Q_{Xt-1} K_{Xt}}{P_t} \phi_{Xt} + \frac{\psi_{P_M}}{2} [\frac{(P_{Mt} - P_{Mt-1})}{P_{Mt}}]^2] \tag{1.30}$$

[17]Implicitly we are assuming that the foreign exporter does not use imports or home non-traded goods in order to pay the costs of price adjustment, but uses foreign goods (either non-traded or goods not consumed by the home country). This is to keep the notation more simple. We found that the results are identical if we assume foreign price adjustment costs must be paid in domestic imports and domestic non-traded goods

Total demand for import goods (necessary to compute the foreign price adjustment equation 2.24) is:

$$T_{Mt} = (1-a)(\frac{P_{Mt}}{P_t})^{-\rho}[C_t + I_{Nt} + I_{Xt} + C_t^{Ne} + C_t^{Xe} + \frac{\psi_D}{2}(D_{t+1} - \bar{D})^2 + \frac{\psi_{P_N}}{2}\frac{(P_{Nt} - P_{Nt-1})^2}{P_{Nt-1}^2} + \frac{R_{KNt}Q_{Nt-1}K_{Nt}}{P_t}\phi_{Nt} + \frac{R_{KXt}Q_{Xt-1}K_{Xt}}{P_t}\phi_{Xt} + \frac{\psi_{P_M}}{2}[\frac{(P_{Mt} - P_{Mt-1})}{P_{Mt}}]^2] \quad (1.31)$$

The households labor supply must be divided between the two sectors:

$$H_{Xt} + H_{Nt} = H_t \quad (1.32)$$

Entrepreneur's labor supply is fixed at one for each entrepreneur:

$$H_{Xt}^e = 1 \quad (1.33)$$

$$H_{Nt}^e = 1 \quad (1.34)$$

The economy's aggregate balance of payment condition may be obtained by summing the budget constraint of households and of entrepreneurs in each sector[18] :

$$\begin{aligned} P_tC_t + P_tC_{Nt}^e + P_tC_{Xt}^e + P_t\frac{\psi_D}{2}(D_{t+1} - \bar{D})^2 + S_t(1+i_t^*)(D_t + D_t^e) \\ +P_t\frac{\psi_{P_N}}{2}\frac{(P_{Nt} - P_{Nt-1})^2}{P_{Nt-1}^2} + P_t(\phi_{Nt}R_{Nt}K_{Nt}Q_{Nt-1} + \phi_{Xt}R_{Xt}K_{Xt}Q_{Xt-1}) \\ +P_t(I_{Nt} + I_{Xt}) = P_{Nt}Y_Nt + P_{Xt}Y_{Xt} + S_t(D_{t+1} + D_{t+1}^e) + \Pi_{Mt} \end{aligned} \quad (1.35)$$

The equilibrium of this economy is a collection of 39 sequences of allocation ($W_t, H_t, P_t, i_t, C_t, C_t^{Ne}$, $C_t^{Xe}, D_t, D_t^e, S_t, \Gamma_t, M_t, C_{Nt}, C_{Mt}, P_{Nt}, P_{Xt}, P_{Mt}, H_{Nt}, H_{Xt}, H_{Nt}^e, H_{Xt}^e, W_{Nt}^e, W_{Xt}^e, K_{Nt}, K_{Xt}, I_{Nt}, I_{Xt}$, $R_{Nt}, R_{Xt}, Q_{Nt}, Q_{Xt}, Y_{Nt}, Y_{Xt}, T_{Mt}, MC_{Nt}, R_{KNt}, R_{KXt}, \bar{\omega_{Nt}}, \bar{\omega_{Xt}}, Z_{Nt+1}, Z_{Xt+1}$), satisfying the equilibrium conditions 2.2 of the text, 1.1-1.17, the counterpart of 1.17 for the export sector, 1.19-1.21, 1.22, 1.24 - 1.26, and the counterpart of the four last conditions for the export sector, 1.28 and its counterpart for the export sector, and 1.29-1.35, where we define $D_t^e = D_{Nt} + D_{Xt}$ as the entrepreneurial sector net foreign debt.

[18]Note to obtain 2.33 we must use the definition of capital accumulation 1.15 and 1.16, as well as the optimality conditions of the unfinished capital goods firms in each sector

2 The derivation of the external finance premium

Here we derive the details underlying the external finance premium used in the text. We closely follow the model of BGG in this regard, so our description is kept brief. We focus on the entrepreneur supplying capital to the non-traded sector(the traded sector is exactly analogous).

At the end of period t a continuum of entrepreneurs (indexed by i) need to finance the purchase of new capital K^i_{Nt+1} that will be used in period $t+1$. Assume that each entrepreneur has access to a technology for converting borrowed funds into capital for use in the non-traded firms. Entrepreneurs are subject to idiosyncratic risk however, so that if one unit of funds(in terms of domestic currency) is invested by entrepreneur i, then the return is given by $\omega^i R_{KNt+1}$, where R_{KNt+1} is the gross return of entrepreneurs' capital investment in the non-traded sector, and ω^i follows a log-normal distribution with with mean $-\frac{\sigma^2_\omega}{2}$ and variance σ^2_ω(so that the expected value of ω^i is unity), and is distributed i.i.d. across entrepreneurs and time.

The realization of ω^i can be observed by the entrepreneur but not by the lender. But lenders can discover the true realization at a cost ϕ times the payoff of the investment. Both lenders and entrepreneurs are risk neutral. Standard results then establish that the optimal contract between entrepreneur and lender is a debt contract, whereby the entrepreneur pays a fixed amount $\bar{\omega^i} R_{KNt+1} Q_t K^i_{Nt+1}$ to the lender if $\omega^i > \bar{\omega^i}$. If $\omega^i < \bar{\omega^i}$, the lender monitors the project, the entrepreneur gets nothing, and the lender receives the full proceeds of investment net of monitoring costs. So the expected return to the entrepreneur is

$$R_{KNt+1} Q_{Nt} K^i_{Nt+1} \left[\int_{\bar{\omega}^i_{Nt+1}}^{\infty} \omega^i f(\omega) d\omega - \bar{\omega}^i_{Nt+1} \int_{\bar{\omega}^i_{Nt+1}}^{\infty} f(\omega) d\omega \right] \equiv R_{KNt+1} Q_{Nt} K^i_{Nt+1} A(\bar{\omega}^i_{Nt+1}) \tag{2.36}$$

The expected return to the lender is then

$$R_{KNt+1} Q_{Nt} K^i_{Nt+1} \left[\bar{\omega}^i_{Nt+1} \int_{\bar{\omega}^i_{Nt+1}}^{\infty} f(\omega) d\omega + (1-\mu) \int_0^{\bar{\omega}^i_{Nt}} \omega^i_{Nt+1} f(\omega) d\omega \right] \equiv R_{KNt+1} Q_{Nt} K^i_{Nt+1} B(\bar{\omega}^i_{Nt+1}) \tag{2.37}$$

Then lender must receive a return at least equal to the world opportunity cost, given by $R^*_{t+1} = 1 + i^*_{t+1}$. Thus, the participation constraint of the lender (in terms of the foreign currency) is:

$$\frac{R_{KNt+1} Q_{Nt} K^i_{Nt+1} B(\bar{\omega}^i_{Nt+1})}{S_{t+1}} = \frac{R^*_{t+1}(Q_{Nt} K^i_{Nt+1} - Z^i_{Nt+1})}{S_t} \tag{2.38}$$

An optimal contract chooses the threshold value $\bar{\omega}^i_{Nt+1}$ and K^i_{Nt+1} to solve the following problem:

$$\max E_t \left(R_{KNt+1} Q_{Nt} K^i_{Nt+1} A(\bar{\omega}^i_{Nt+1}) \right) \tag{2.39}$$

subject to the participation constraint 2.38.

Note that the only aggregate uncertainty faced by the entrepreneur and lender is the exchange rate that will prevail when the foreign currency loans must be repaid. And it is assumed that the risk-neutral entrepreneurs bear all the aggregate risk. So the return of the investment R_{KNt+1} and thus the optimal threshold level $\bar{\omega}^i_{Nt+1}$ will be state contingent on the realizations of the exchange rate and the participation constraint will hold with equality, at every possible state ex post.

The two first order condition implied by the contract is then:

$$E_t \left[R_{KNt+1} Q_{Nt} A(\bar{\omega}^i_{Nt+1}) \right] + E_t \left[\lambda_{t+1} \frac{R_{KNt+1} Q_{Nt} A(\bar{\omega}^i_{Nt+1})}{S_{t+1}} - \frac{R^*_{t+1} Q_{Nt}}{S_t} \right] = 0 \tag{2.40}$$

$$\lambda_{t+1}(\theta) = -\frac{\pi(\theta) A'(\bar{\omega}^i_{Nt+1}(\theta)) S_{t+1}(\theta)}{B'(\bar{\omega}^i_{Nt+1}(\theta))} \tag{2.41}$$

where $\theta \in \Theta$ is a state of the world, $\pi(\theta)$ is the probability of state θ and λ_{t+1} is the Lagrange multiplier associated with the participation constraint. Substitute 2.41 into 2.40, we get:

$$E_t \left\{ R_{KNt+1} \left[\frac{A'(\bar{\omega}^i_{Nt+1})}{B'(\bar{\omega}^i_{Nt+1})} B(\bar{\omega}^i_{Nt+1}) - A(\bar{\omega}^i_{Nt+1}) \right] \right\} = E_t \left[\frac{A'(\bar{\omega}^i_{Nt+1})}{B'(\bar{\omega}^i_{Nt+1})} \frac{S_{t+1}}{S_t} R^*_{t+1} \right] \tag{2.42}$$

Since ω^i is i.i.d across entrepreneurs, every entrepreneur actually faces the same financial contract, so we could drop the superscript i. Rearranging 2.42, we could get 1.22.

The entrepreneurs are assumed to die at any time period with probability $(1-\nu)$. Thus, at any given period, a fraction $(1-\nu)$ of entrepreneurial wealth is consumed. So the consumption of entrepreneurs in the non-traded sector is given by 1.25. And the net wealth Z_{Nt+1} is given by:

$$Z_{Nt+1} = \nu R_{KNt} Q_{Nt-1} K_{Nt} A(\bar{\omega}_{Nt}) + W^e_{Xt} \tag{2.43}$$

Use the fact that $B(\bar{\omega}) = 1 - A(\bar{\omega}) - \mu \int_0^{\bar{\omega}} \omega f(\omega) d\omega$ and imposing the participation constraint, we get 1.26.

3 Derivation of $A(\cdot)$, $A'(\cdot)$, $B(\cdot)$ and $B'(\cdot)$

We know that:

$$A(\bar{\omega}) = \int_{\bar{\omega}}^{\infty} \omega f(\omega) d\omega - \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega \tag{3.44}$$

$$B(\bar{\omega}) = \bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega + (1-\mu) \int_{0}^{\bar{\omega}} \omega f(\omega) d\omega \tag{3.45}$$

If ω_t^i is log-normally distributed with mean $-\frac{\sigma_\omega^2}{2}$ and variance σ_ω^2, we know that

$$E(\omega) = \int_{-\infty}^{\infty} \omega f(\omega) d\omega = 1 \tag{3.46}$$

where the density function $f(\omega)$ is given by:

$$f(\omega) = \frac{1}{\sigma_\omega \omega \sqrt{2\pi}} \exp\left\{ -\frac{(\ln\omega + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} \tag{3.47}$$

Then we may write

$$\begin{aligned}
\int_{\bar{\omega}}^{\infty} \omega f(\omega) d\omega &= \int_{\ln\bar{\omega}}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp\left\{ -\frac{(y + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} \exp(y) dy \\
&= \int_{\ln\bar{\omega}}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp\left\{ -\frac{(y - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} dy \\
&= \frac{1}{\sqrt{\pi}} \int_{\ln\bar{\omega}}^{\infty} \exp\left\{ -\frac{(y - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} d\left(\frac{y - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right) \\
&= \frac{1}{2} erfc\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right)
\end{aligned} \tag{3.48}$$

where $erfc(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} dt$ is the "complementary error function".

And similarly,

$$\begin{aligned}
\bar{\omega} \int_{\bar{\omega}}^{\infty} f(\omega) d\omega &= \bar{\omega} \int_{\bar{\omega}}^{\infty} \frac{1}{\sigma_\omega \omega \sqrt{2\pi}} \exp\left\{ -\frac{(\ln\omega + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} d\omega \\
&= \bar{\omega} \int_{\bar{\omega}}^{\infty} \frac{1}{\sigma_\omega \sqrt{2\pi}} \exp\left\{ -\frac{(\ln\omega + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} d\ln\omega \\
&= \bar{\omega} \int_{\ln\bar{\omega}}^{\infty} \frac{1}{\sqrt{\pi}} \exp\left\{ -\frac{(\ln\omega + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2} \right\} d\left(\frac{\ln\omega + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right) \\
&= \frac{\bar{\omega}}{2} erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega} \right)
\end{aligned} \tag{3.49}$$

So we get:

$$A(\bar{\omega}) = \frac{1}{2}erfc\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right) - \frac{\bar{\omega}}{2}erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right) \tag{3.50}$$

Then we may write

$$\begin{aligned}\int_0^{\bar{\omega}} \omega f(\omega)d\omega &= \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\ln\bar{\omega}} \exp\left\{-\frac{(y-\frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2}\right\} d(\frac{y-\frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}) \\ &= \frac{1}{2}\left[1 + erf\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right)\right] \end{aligned} \tag{3.51}$$

$$B(\bar{\omega}) = \frac{\bar{\omega}}{2}erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right) + (1-\mu)\frac{1}{2}\left[1 + erf\left(\frac{\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right)\right] \tag{3.52}$$

where $erf(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}dt$ is the "error function".

Therefore, it can be easily derived that:

$$A'(\bar{\omega}) = -\frac{1}{\sqrt{2\pi}\sigma_\omega}\left[\frac{1}{\bar{\omega}}\exp\left(-\frac{(\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2}\right) - \exp\left(-\frac{(\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2}\right)\right] - \frac{1}{2}erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right) \tag{3.53}$$

But we can prove that

$$\begin{aligned}\frac{1}{\bar{\omega}}\exp\left(-\frac{(\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2}\right) &= \exp[-\ln(\bar{\omega})]\exp\left(-\frac{(\ln(\bar{\omega}) - \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2}\right) \\ &= \exp\left(-\frac{(\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2}\right)\end{aligned} \tag{3.54}$$

Therefore,

$$A'(\bar{\omega}) = -\frac{1}{2}erfc\left(\frac{\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2}}{\sqrt{2}\sigma_\omega}\right) \tag{3.55}$$

Note that $E(\omega) = 1$, so $B(\bar{\omega}) = 1 - A(\bar{\omega}) - \mu\int_0^{\bar{\omega}} \omega f(\omega)d\omega$, thus

$$B'(\bar{\omega}) = -A'(\bar{\omega}) - \frac{\mu}{\sqrt{2\pi}\sigma_\omega}\exp\left(-\frac{(\ln(\bar{\omega}) + \frac{\sigma_\omega^2}{2})^2}{2\sigma_\omega^2}\right) \tag{3.56}$$

4 Computing the Consumption Equivalent Welfare Measures

This section gives the details of the derivation of the consumption equivalent comparisons ϵ. First take the model without entrepreneurs. For monetary policy regime r, the expected utility can be written as:

$$V^r = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{r\,(1-\sigma)}}{1-\sigma} - \eta \frac{H_t^{r\,(1+\psi)}}{1+\psi} \right) \tag{4.57}$$

where $\{C_t^r\}$ and $\{H_t^r\}$ are the stream of the consumption and labour supply under policy regime r. To compare across different regimes, we may define C^τ and H^τ as the permanent(annuity) consumption and labor supply associate with regime τ such that

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{r\,(1-\sigma)}}{1-\sigma} - \eta \frac{H_t^{r\,(1+\psi)}}{1+\psi} \right) = \sum_{t=0}^{\infty} \beta^t \left(\frac{C^{r\,(1-\sigma)}}{1-\sigma} - \eta \frac{H^{r\,(1+\psi)}}{1+\psi} \right) \tag{4.58}$$

Thus, the expected utility under regime r is given by

$$V^r = \frac{C^{r\,(1-\sigma)}}{(1-\sigma)(1-\beta)} - \eta \frac{H^{r\,(1+\psi)}}{(1+\psi)(1-\beta)} \tag{4.59}$$

Similarly, the expected utility under monetary policy regime s can be written as:

$$V^s = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{s(1-\sigma)}}{1-\sigma} - \eta \frac{H_t^{s(1+\psi)}}{1+\psi} \right) = \frac{C^{s(1-\sigma)}}{(1-\sigma)(1-\beta)} - \eta \frac{H^{s(1+\psi)}}{(1+\psi)(1-\beta)} \tag{4.60}$$

ϵ is defined as the fraction of permanent consumption that a consumer in an economy governed by monetary policy r would be willing to give up in order to make her indifferent between this and an economy governed by monetary policy s, Thus, ϵ can be derived from the following equality

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{[(1-\epsilon)C^r]^{(1-\sigma)}}{1-\sigma} - \eta \frac{H^{r(1+\psi)}}{1+\psi} \right) = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C^{s(1-\sigma)}}{1-\sigma} - \eta \frac{H^{s(1+\psi)}}{1+\psi} \right) \tag{4.61}$$

Or

$$\frac{[(1-\epsilon)C^r]^{(1-\sigma)}}{(1-\sigma)(1-\beta)} - \eta \frac{H^{r(1+\psi)}}{(1+\psi)(1-\beta)} = \frac{C^{s(1-\sigma)}}{(1-\sigma)(1-\beta)} - \eta \frac{H^{s(1+\psi)}}{(1+\psi)(1-\beta)} \tag{4.62}$$

Define $\eta \frac{H^{\tau(1+\psi)}}{(1+\psi)(1-\beta)}$ as V_h^τ, the disutility of labor under regime τ, we may get:

$$\begin{aligned} \frac{[(1-\epsilon)C^r]^{(1-\sigma)}}{(1-\sigma)(1-\beta)} &= V^s + V_h^r \\ \Rightarrow \epsilon &= 1 - \frac{[(V^s + V_h^r)(1-\sigma)(1-\beta)]^{\frac{1}{1-\sigma}}}{C^r} \end{aligned} \tag{4.63}$$

From Equation 4.59, we may get

$$C^r = [(V^r + V_h^r)(1-\sigma)(1-\beta)]^{\frac{1}{1-\sigma}} \tag{4.64}$$

Thus

$$\epsilon = 1 - \left(\frac{V^s + V_h^r}{V^r + V_h^r}\right)^{\frac{1}{1-\sigma}} \tag{4.65}$$

For the economy with entrepreneurs, ϵ is defined as the fraction of permanent consumption that must be offered both to households and entrepreneurs so as to make them indifferent between the two regimes

$$\frac{[(1-\epsilon)C^r]^{(1-\sigma)}}{(1-\sigma)(1-\beta)} - \eta\frac{H^{r(1+\psi)}}{(1+\psi)(1-\beta)} + \frac{(1-\epsilon)C^{re}}{1-\beta} = \frac{C^{s(1-\sigma)}}{(1-\sigma)(1-\beta)} - \eta\frac{H^{s(1+\psi)}}{(1+\psi)(1-\beta)} + \frac{C^{se}}{1-\beta} \equiv V^s \tag{4.66}$$

where $C^{\tau e}$ is the permanent consumption of entrepreneurs under regime τ.

If we define $V_e^\tau = \frac{C^{\tau e}}{1-\beta}$ as the expected utility for entrepreneurs under regime τ, we may derive ϵ analogously:

$$\frac{[(1-\epsilon)C^r]^{(1-\sigma)}}{(1-\sigma)(1-\beta)} + (1-\epsilon)V_e^r = V^s + V_h^r \tag{4.67}$$

Since $\frac{C^{r(1-\sigma)}}{(1-\sigma)(1-\beta)} = V^r + V_h^r - V_e^r$, we may derive ϵ implicitly from the following equation:

$$(1-\epsilon)^{1-\sigma}(V^r + V_h^r - V_e^r) + (1-\epsilon)V_e^r = V^s + V_h^r \tag{4.68}$$

5 The model without entrepreneurs

The comparison economy without private information or an entrepreneurial sector is identical to the set-up we have described, except that capital is accumulated directly by households without any external finance constraint. Here we simply list the equations used to solve this economy. They are exactly analogous to those of the previous model, except in the details of the determination of aggregate capital, and the absence of entrepreneurial consumption and wealth dynamics. They are:

$$W_t = \eta H_t^\psi P_t C_t^\sigma \tag{5.1}$$

$$\frac{1}{1+i_{t+1}} = \beta E_t\left(\frac{C_t^\sigma P_t}{C_{t+1}^\sigma P_{t+1}}\right) \tag{5.2}$$

$$\frac{1}{1+i^*_{t+1}}\left[1-\frac{\psi_D P_t}{S_t}(D_{t+1}-\bar{D})\right]=\beta E_t\left\{\frac{C_t^\sigma P_t}{C_{t+1}^\sigma P_{t+1}}\frac{S_{t+1}}{S_t}\right\} \tag{5.3}$$

$$\frac{M_t}{P_t}=\frac{\chi^{\frac{1}{\varepsilon}}C_t^{\frac{\sigma}{\varepsilon}}}{\left(1-\frac{1}{1+i_{t+1}}\right)^{\frac{1}{\varepsilon}}} \tag{5.4}$$

$$P_t=(aP_{Nt}^{1-\rho}+(1-a)P_{Mt}^{1-\rho})^{\frac{1}{1-\rho}} \tag{5.5}$$

$$W_t=MC_{Nt}(1-\alpha)\frac{Y_{Nt}}{H_{Nt}} \tag{5.6}$$

$$R_{Nt}=MC_{Nt}\alpha\frac{Y_{Nt}}{K_{Nt}} \tag{5.7}$$

$$Y_{Nt}=A_N K_{Nt}^{\alpha}H_{Nt}^{(1-\alpha)} \tag{5.8}$$

$$W_t=P_{Xt}(1-\gamma)\frac{Y_{Xt}}{H_{Xt}} \tag{5.9}$$

$$R_{Xt}=P_{Xt}\gamma\frac{Y_{Xt}}{K_{Xt}} \tag{5.10}$$

$$Y_{Xt}=A_X K_{Xt}^{\gamma}H_{Xt}^{(1-\gamma)} \tag{5.11}$$

$$P_{Nt}=\frac{\lambda}{\lambda-1}MC_{Nt}-\frac{\psi_{P_N}}{\lambda-1}\frac{P_t}{Y_{Nt}}\frac{P_{Nt}}{P_{Nt-1}}\left(\frac{P_{Nt}}{P_{Nt-1}}-1\right)+\frac{\psi_{P_N}}{\lambda-1}E_t\left[\Gamma_{t+1}\frac{P_{t+1}}{Y_{Nt}}\frac{P_{Nt+1}}{P_{Nt}}\left(\frac{P_{Nt+1}}{P_{Nt}}-1\right)\right] \tag{5.12}$$

$$P_{Mt}=\frac{\lambda}{\lambda-1}SP_{Mt}^*-\frac{\psi_{P_M}}{\lambda-1}\frac{P_t}{T_{Mt}}\frac{P_{Mt}}{P_{Mt-1}}\left(\frac{P_{Mt}}{P_{Mt-1}}-1\right)+\frac{\psi_{P_M}}{\lambda-1}E_t\left[\Gamma_{t+1}\frac{P_{t+1}}{T_{Mt}}\frac{P_{Mt+1}}{P_{Mt}}\left(\frac{P_{Mt+1}}{P_{Nt}}-1\right)\right] \tag{5.13}$$

$$P_{Xt}=S_tP_{Xt}^* \tag{5.14}$$

$$Q_{Xt}=\frac{P_t}{1-\psi_I(\frac{I_{Xt}}{K_{Xt}}-\delta)} \tag{5.15}$$

$$Q_{Nt}=\frac{P_t}{1-\psi_I(\frac{I_{Nt}}{K_{Nt}}-\delta)} \tag{5.16}$$

$$K_{Xt+1}=\left[\frac{I_{Xt}}{K_{Xt}}-\frac{\psi_I}{2}\left(\frac{I_{Xt}}{K_{Xt}}-\delta\right)^2\right]K_{Xt}+(1-\delta)K_{Xt} \tag{5.17}$$

$$K_{Nt+1}=\left[\frac{I_{Nt}}{K_{Nt}}-\frac{\psi_I}{2}\left(\frac{I_{Nt}}{K_{Nt}}-\delta\right)^2\right]K_{Nt}+(1-\delta)K_{Nt} \tag{5.18}$$

$$E_t\left[\frac{R_{KNt+1}}{C_{t+1}^\sigma P_{t+1}}\right]=\frac{1}{C_t^\sigma P_t} \tag{5.19}$$

$$R_{KNt+1} = \frac{R_{Nt+1} + \left[1 - \delta + \psi_I(\frac{I_{Nt+1}}{K_{Nt+1}} - \delta)\frac{I_{Nt+1}}{K_{Nt+1}} - \frac{\psi_I}{2}(\frac{I_{Nt+1}}{K_{Nt+1}} - \delta)^2\right] Q_{Nt+1}}{Q_{Nt}} \tag{5.20}$$

$$E_t\left[\frac{R_{KXt+1}}{C_{t+1}^{\sigma} P_{t+1}}\right] = \frac{1}{C_t^{\sigma} P_t} \tag{5.21}$$

$$R_{KXt+1} = \frac{R_{Xt+1} + \left[1 - \delta + \psi_I(\frac{I_{Xt+1}}{K_{Xt+1}} - \delta)\frac{I_{Xt+1}}{K_{Xt+1}} - \frac{\psi_I}{2}(\frac{I_{Xt+1}}{K_{Xt+1}} - \delta)^2\right] Q_{Xt+1}}{Q_{Xt}} \tag{5.22}$$

$$Y_{Nt} = a(\frac{P_{Nt}}{P_t})^{-\rho}[C_t + I_{Nt} + I_{Xt} + \frac{\psi_D}{2}(D_{t+1} - \bar{D})^2 + \frac{\psi_{P_M}}{2}(\frac{P_{Mt}}{P_{Mt-1}} - 1)^2 + \frac{\psi_{P_N}}{2}(\frac{P_{Nt}}{P_{Nt-1}} - 1)^2] \tag{5.23}$$

$$T_{Mt} = (1 - a)(\frac{P_{Mt}}{P_t})^{-\rho}[C_t + I_{Nt} + I_{Xt} + \frac{\psi_D}{2}(D_{t+1} - \bar{D})^2 + \frac{\psi_{P_M}}{2}(\frac{P_{Mt}}{P_{Mt-1}} - 1)^2 + \frac{\psi_{P_N}}{2}(\frac{P_{Nt}}{P_{Nt-1}} - 1)^2] \tag{5.24}$$

$$H_{Xt} + H_{Nt} = H_t \tag{5.25}$$

$$S_t(1 + i_t^*)D_t - S_t D_{t+1} = P_{Xt} Y_{Xt} - S_t P_{Mt}^* T_{Mt} \tag{5.26}$$

$$1 + i_{t+1} = \left(\frac{P_{Nt}}{P_{Nt-1}} \frac{1}{\bar{\pi_n}}\right)^{\mu_{\pi_n}} \left(\frac{P_t}{[a(P_{Nt-1})^{1-\rho} + (1-a)(P_{Lt-1})^{1-\rho}]^{\frac{1}{1-\rho}}} \frac{1}{\bar{\pi}}\right)^{\mu_\pi} \left(\frac{S_t}{\bar{S}}\right)^{\mu_S} (1 + \bar{i}) \tag{5.27}$$