Technical Appendix

Not to be published

A Households and firms

A.1 Consumption and price index

The aggregate consumption in the home country is a CES aggregate of the home and foreign composite consumption goods:

$$C = \frac{C_h^n C_f^{1-n}}{n^n(1-n)^{1-n}} \tag{A.1}$$

where the home and foreign composite goods are give by

$$C_h = [n^{-\frac{1}{\lambda}} \int_0^n C_h(i)^{\frac{\lambda-1}{\lambda}} di]^{\frac{\lambda}{\lambda-1}} \qquad C_f = [(1-n)^{-\frac{1}{\lambda}} \int_n^1 C_f(i)^{\frac{\lambda-1}{\lambda}} di]^{\frac{\lambda}{\lambda-1}} \tag{A.2}$$

Price indices, associated with each consumption aggregate, are defined as the minimum nominal cost of purchasing one unit the relevant basket. Thus, we have

$$P = P_h^n P_f^{1-n} \tag{A.3}$$

$$P_h = [\frac{1}{n} \int_0^n P_h(i)^{1-\lambda} di]^{\frac{1}{1-\lambda}}, \quad P_f = [\frac{1}{1-n} \int_n^1 P_f(i)^{1-\lambda} di]^{\frac{1}{1-\lambda}} \tag{A.4}$$

Taking prices for the individual goods and the composite home and foreign goods as given, we can derive the optimal demand function for each individual goods and the composite goods[1] :

$$C_h = n\frac{PC}{P_h}, \qquad C_f = (1-n)\frac{PC}{P_f} \tag{A.5}$$

$$C_h(i) = \frac{1}{n}[\frac{P_h(i))}{P_h}]^{-\lambda} C_h, \qquad C_f(i) = \frac{1}{1-n}[\frac{P_f(i))}{P_f}]^{-\lambda} C_f. \tag{A.6}$$

A.2 Firm

The optimization problem of each firm is to choose the currency in which the firm sets its export goods and then to preset prices to maximize its discounted expected profits, taking the individual demand function as given.

[1]The consumer just allocates a given level of consumption aggregate among the differentiated goods.

A.2.1 The optimal price given the currency of pricing

If a home firm i follows PCP, it will set both the domestic price and export price in home currency (the currency of producer), then the optimization problem for firm i is:

$$\max_{P_{hh},P_{hf}} E[d\pi(i)] = \max_{P_{hh},P_{hf}} E[d((P_{hh}(i) - \frac{W}{\theta})X_h(i) + (P_{hf}(i) - \frac{W}{\theta})X_h^*(i))] \tag{A.7}$$

where $d = P^{-1}C^{-\rho}$ is the stochastic discount factor[2], $X_h(i) = nC_h(i)$ is the total sales of firm i to home residents and $X_h^*(i) = (1-n)C_h^*(i)$ is the total sales to foreign residents. $P_{hh}(i)$ is the price of goods sold to domestic consumers, $P_{hf}(i)$ is the price of export goods, denominated in home currency. [3]

If firm i follow LCP, it will set domestic price in domestic currency and export price in foreign currency (the currency of buyer), then the optimization problem for firm i is:

$$\max_{P_{hh},P_{hf}^*} E[d\pi(i)] = \max_{P_{hh},P_{hf}} E[d((P_{hh}(i) - \frac{W}{\theta})X_h(i) + (SP_{hf}^*(i) - \frac{W}{\theta})X_h^*(i))] \tag{A.8}$$

where $P_{hf}^*(i)$ is the price of export goods, denominated in foreign currency.

Under a symmetric equilibrium, using the risk-sharing condition, the labor supply function and its foreign equivalent, we can simplify the home and foreign firms' optimal pricing schedule under different pricing specifications. They are listed in Table 1.

[2] All firms are assumed to be owned by the domestic households. Hence, we assume that the firm maximizes its value, evaluated using the marginal utility of its owner in each state of the world. In fact, since we have assumed a complete set of state-contingent nominal assets, this is equivalent to firms choosing a price to maximize the value of profits summed across states, weighted by nominal state prices.

[3] In the symmetric PCP case, that is, if all the home and foreign firms choose PCP, then $P_{hh}(i)$ equals $P_{hf}(i)$.

Table 1: Three optimal pricing policies[a]

Price	{PCP, PCP}	{LCP, LCP}	{PCP, LCP}
P_{hh}	$\hat{\lambda}\frac{E[\frac{WC^{1-\rho}}{\theta}]}{E[C^{1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{WC^{1-\rho}}{\theta}]}{E[C^{1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{WC^{1-\rho}}{\theta}]}{E[C^{1-\rho}]}$
P^*_{hf}	$\frac{P_{hh}}{S}$	$\hat{\lambda}\frac{E[\frac{WC^{*1-\rho}S^{-1}}{\theta}]}{E[C^{*1-\rho}]}$	$\hat{\lambda}\frac{S^{-1}E[\frac{WC^{*1-\rho}}{\theta}]}{E[C^{*1-\rho}]}$
P^*_{ff}	$\hat{\lambda}\frac{E[\frac{W^*C^{*1-\rho}}{\theta^*}]}{E[C^{*1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{W^*C^{*1-\rho}}{\theta^*}]}{E[C^{*1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{W^*C^{*1-\rho}}{\theta^*}]}{E[C^{*1-\rho}]}$
P_{fh}	SP^*_{ff}	$\hat{\lambda}\frac{E[\frac{SW^*C^{1-\rho}}{\theta^*}]}{E[C^{1-\rho}]}$	$\hat{\lambda}\frac{E[\frac{SW^*C^{1-\rho}}{\theta^*}]}{E[C^{1-\rho}]}$

[a]Where $\hat{\lambda} = \frac{\lambda}{\lambda-1}$ is the markup.

A.2.2 Endogenous currency of pricing

For each firm i in the home country selling a differentiated good to the foreign market, it faces a downward-sloping demand function

$$X(P^*_{hf}(i)) = (\frac{P^*_{hf}(i)}{P^*_{hf}})^{-\lambda}\frac{P^*}{P^*_{hf}}C^* \tag{A.9}$$

where $P^*_{hf}(i)$ is the price that the foreign consumer pays for the home goods i. P^*_{hf} is the price index for all home goods sold on the foreign market, and P^* is the foreign country consumer price index. Without loss of generality, let $P^*_{hf}(i)$, P^*_{hf} and P^* be denominated in foreign currency.

If home firm sets its price under PCP, then the expected discounted profits are

$$E\Pi^{PCP} = E[d(P^{PCP}_{hf}(i) - MC)(\frac{P^{PCP}_{hf}(i)}{SP^*_{hf}})^{-\lambda}\frac{P^*}{P^*_{hf}}C^*] \tag{A.10}$$

where d is the stochastic discount factor. It is equal to the marginal utility of consumption, as all home firms are assumed to be owned by home households, $MC = \frac{W}{\theta}$ is the marginal cost of home firms. If the home firm set its price under LCP, then the expected discounted profits are

$$E\Pi^{LCP} = E[d(SP^{LCP}_{hf}(i) - MC)(\frac{P^{LCP}_{hf}(i)}{P^*_{hf}})^{-\lambda}\frac{P^*}{P^*_{hf}}C^*] \tag{A.11}$$

The home country firm will set its price in its own currency if the expected profit differential is positive. That is, it follows PCP if and only if

$$E\Pi^{PCP} - E\Pi^{LCP} > 0 \tag{A.12}$$

The profit-maximization price for the firm, under PCP and LCP, respectively, may easily be derived as follows: [4]

$$P^{PCP}(i) = \hat{\lambda}\frac{E(MCS^{\lambda}Z)}{E(S^{\lambda}Z)}, \qquad P^{PCP}(i) = \hat{\lambda}\frac{E(MCZ)}{E(SZ)} \tag{A.13}$$

Where $Z = dP^{\lambda-\theta}P^{*\theta}Y^{*}$. Using these solutions, we can computer the expected discounted profits as:

$$E\Pi^{PCP} = \widetilde{\lambda}[E(S^{\lambda}Z)]^{\lambda}[E(S^{\lambda}ZMC)]^{1-\lambda} \tag{A.14}$$

$$E\Pi^{LCP} = \widetilde{\lambda}[E(SZ)]^{\lambda}[E(ZMC)]^{1-\lambda} \tag{A.15}$$

where $\widetilde{\lambda} = \frac{1}{\lambda-1}(\frac{\lambda}{\lambda-1})^{-\lambda}$. Now using the second order approximation, we have

$$\begin{aligned} E\Pi^{PCP} \approx \ln\sum + [\frac{1}{2}var(\ln Z) + \frac{\lambda^2}{2}var(\ln S) + \frac{1-\lambda}{2}var(\ln MC)] + \\ \{\lambda cov(\ln Z, \ln S) + \lambda(1-\lambda)cov(\ln MC, \ln S) + (1-\lambda)cov(\ln Z, \ln MC)\} \end{aligned} \tag{A.16}$$

$$\begin{aligned} E\Pi^{LCP} \approx \ln\sum + [\frac{1}{2}var(\ln Z) + \frac{\lambda}{2}var(\ln S) + \frac{1-\lambda}{2}var(\ln MC)] + \\ \{\lambda cov(\ln Z, \ln S) + (1-\lambda)cov(\ln Z, \ln MC)\} \end{aligned} \tag{A.17}$$

where $\sum = \tilde{\lambda}\exp[E(\ln Z)]\exp[\lambda E(\ln S)\exp[(1-\lambda)E(\ln W)]$.

From now on, let $x = \ln(X)$. Comparing (A.16) and (A.17), we can establish the condition for the decision of currency choice. Home firms wil follow PCP whenever

$$\frac{1}{2}\sigma_s^2 - cov(mc, s) > 0 \tag{A.18}$$

The equivalent condition for the foreign firms is

$$\frac{1}{2}\sigma_s^2 + cov(mc^*, s) > 0 \tag{A.19}$$

[4] These optimal price schedules are the general form under PCP or LCP.

We define the left-hand side of A.18 and A.19 as Ω and Ω^*. Using the solution that $S = \Gamma\frac{M}{M^*}$ and $W = \frac{\eta}{\chi}M$, we can express Ω and Ω^* as functions of the underling monetary policy parameters.

If the monetary authorities follow the unrestricted money rules, we can compute

$$\sigma_s^2 = (a_1 - b_1)^2\sigma_u^2 + (a_2 - b_2)^2\sigma_{u^*}^2 \tag{A.20}$$

$$cov(mc, s) = (a_1 - b_1)(a_1 - 1)\sigma_u^2 + (a_2 - b_2)a_2\sigma_{u^*}^2 \tag{A.21}$$

$$cov(mc^*, s) = (a_1 - b_1)b_1\sigma_u^2 + (a_2 - b_2)(b_2 - 1)\sigma_{u^*}^2 \tag{A.22}$$

Thus,

$$\Omega = \frac{1}{2}(a_1 - b_1)(2 - a_1 - b_1)\sigma_u^2 + \frac{1}{2}(a_2 - b_2)(-a_2 - b_2)\sigma_{u^*}^2 \tag{A.23}$$

$$\Omega^* = \frac{1}{2}(a_1 - b_1)(a_1 + b_1)\sigma_u^2 + \frac{1}{2}(a_2 - b_2)(a_2 + b_2 - 2)\sigma_{u^*}^2 \tag{A.24}$$

Similarly, if monetary authorities choose restricted money rules, we have

$$\Omega = \frac{1}{2}a(2 - a)\sigma_u^2 + \frac{1}{2}b^2\sigma_{u^*}^2 \tag{A.25}$$

$$\Omega^* = \frac{1}{2}a^2\sigma_u^2 + \frac{1}{2}b(2 - b)\sigma_{u^*}^2 \tag{A.26}$$

B Equilibrium

In this section, we will discuss the existence of equilibrium under different monetary policy rules. In equilibrium, all firms within one country follow the same pricing strategy, so there are four cases where all firms in both countries follow PCP, where all firms in both countries follow LCP, or where home firms follow PCP and foreign firms follow LCP (and vice versa). Given the timing in Figure 1 of the paper, for each case, we first compute the mean and variances of (log) consumption for both countries as functions of the policy parameters, and use them to find out the expected utility of home and foreign households. We then characterize the outcome of an international monetary game, where monetary authorities take the currency of pricing decision as given. Finally, in order to ensure that any given set of optimal policy rules represents an equilibrium, they must be consistent with the currency of pricing decision made by firms.

B.1 Multiple equilibria under unrestricted monetary policy rules

B.1.1 Case 1 {PCP, PCP}

Under the PCP specification, since purchasing power parity $P = SP^*$ holds, the across country risk-sharing condition $\Gamma PC^\rho = SP^*C^{*\rho}$ implies [5]

$$C = C^* \tag{B.1}$$

The expected utility for home country, EU can be calculated by the following procedures.

Solving for Ec Given the price index in the home country, $P = P_{hh}^n(SP_{ff}^*)^{1-n}$ and the pricing equations in Table 1, we have

$$P = \hat{\lambda}\frac{[E(\frac{WC^{1-\rho}}{\theta})]^n}{[E(C^{1-\rho})]^n}\frac{[E(\frac{W^*C^{*1-\rho}}{\theta^*})]^{1-n}S^{1-n}}{[E(C^{*1-\rho})]^{1-n}} \tag{B.2}$$

Using B.1, the labor supply function $W = \eta PC^\rho$ and its foreign equivalence, taking out the predetermined price terms P_{hh} and P_{ff}^* in both sides, we may rewrite B.2 as

$$1 = \hat{\lambda\eta}\frac{[E(\frac{CS^{1-n}}{\theta})]^n[E(\frac{CS^{-n}}{\theta^*})]^{1-n}}{E(C^{1-\rho})} \tag{B.3}$$

Using the fact that the model solution is log linear and the shocks are log normal, and taking logs, we may solve for the value of the expected (log) consumption:

$$\begin{aligned} Ec = -\frac{1}{\rho}\ln(\hat{\lambda\eta}) - \frac{2-\rho}{2}\sigma_c^2 - \frac{n(1-n)}{2\rho}\sigma_s^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} \\ + \frac{(n\sigma_{cu} + (1-n)\sigma_{cu^*})}{\rho} + n(1-n)\frac{\sigma_{su} - \sigma_{su^*}}{\rho} \end{aligned} \tag{B.4}$$

Solving for EL and EU Under the PCP specification, the goods market clearing condition in the home country is

$$\theta L = n\frac{PC}{P_{hh}} + (1-n)\frac{P^*C^*}{\frac{P_{hh}}{S}} \tag{B.5}$$

We can simplify it as $\theta L = \frac{PC}{P_{hh}}$ as PPP holds under the {PCP, PCP} configuration. Now using the labor supply function, and the pricing equations in Table 1, we can obtain:

$$EL = \frac{EC^{1-\rho}}{\hat{\lambda\eta}} \tag{B.6}$$

[5] The appendix for Devereux and Engel (2003) show that $\Gamma = 1$ will hold in a symmetric equilibrium of monetary policy game, for both PCP and LCP price setting regimes.

Thus, we can express the expected utility of the home representative consumer as

$$EU = \frac{EC^{1-\rho}}{1-\rho}\{\frac{\lambda - (1-\rho)(\lambda - 1)}{\lambda}\} \tag{B.7}$$

Since we have $EC^{1-\rho} = \exp(1-\rho)(Ec + \frac{1-\rho}{2}\sigma_c^2)$, we may rewrite EU as:

$$\begin{aligned} EU = \Theta \exp(1-\rho)[-\frac{1}{2}\sigma_c^2 - \frac{n(1-n)}{2\rho}\sigma_s^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} \\ + \frac{(n\sigma_{cu} + (1-n)\sigma_{cu^*})}{\rho} + n(1-n)\frac{\sigma_{su} - \sigma_{su^*}}{\rho}] \end{aligned} \tag{B.8}$$

Where $\Theta = \frac{\lambda-(1-\rho)(\lambda-1)}{(1-\rho)\lambda}(\lambda\eta)^{\frac{\rho-1}{\rho}} < 0$ is a constant function of parameters.

Solving for monetary policy parameters In log terms, we may write the exchange rate and the consumption as:

$$s - E(s) = m - m^* \tag{B.9}$$

$$c - Ec = \frac{nm + (1-n)m^*}{\rho} \tag{B.10}$$

Under the unrestricted monetary policy rules, from B.9 and B.10, we can derive the following variance and covariance terms:

$$\sigma_s^2 = (a_1 - b_1)^2\sigma_u^2 + (a_2 - b_2)^2\sigma_{u^*}^2, \quad \sigma_c^2 = \frac{(na_1 + (1-n)b_1)^2\sigma_u^2 + (na_2 + (1-n)b_2)^2\sigma_{u^*}^2}{\rho^2}, \tag{B.11}$$

$$\sigma_{cu} = \frac{(na_1 + (1-n)b_1)\sigma_u^2}{\rho}, \qquad \sigma_{cu^*} = \frac{(na_2 + (1-n)b_2)\sigma_{u^*}^2}{\rho} \tag{B.12}$$

$$\sigma_{su} = (a_1 - b_1)\sigma_u^2, \qquad \sigma_{su^*} = (a_2 - b_2)\sigma_{u^*}^2 \tag{B.13}$$

Substituting the above terms into B.8, we may express the expected utility of the home country as a function of the feedback parameters. Note that the expected utility of the foreign country is identical to that of the home country since $C = C^*$. Thus, the optimal monetary rules can be determined by solving the following Nash monetary game.

$$\max_a EU(a, b^n) \tag{B.14}$$

$$\max_b EU^*(a^n, b) \tag{B.15}$$

The solution to this game is

$$a = [1, 0], \qquad b = [0, 1] \tag{B.16}$$

Checking the currency decision Substituting the policy parameters back into A.23 and A.24, we have

$$\Omega = \Omega^* = \frac{1}{2}(\sigma_u^2 + \sigma_{u^*}^2) > 0 \tag{B.17}$$

So all the firms will follow PCP given these monetary policies. That is, all firms choosing PCP is an equilibrium. The optimal monetary policy associated with this equilibrium ensures the flexible exchange rate($s - E(s) = u - u^*$), and the expected utility associated with this equilibrium is

$$EU = \Theta \exp\{(1-\rho)\frac{(1-\rho)[n^2\sigma_u^2 + (1-n)^2\sigma_{u^*}^2]}{2\rho^2}\} \tag{B.18}$$

B.1.2 Case 2 {LCP, LCP}

Under the LCP specification, the purchasing power parity will not generally hold, so the home and foreign consumption will differ.

Solving for Ec and Ec^* In this case, the price level in each country is completely predetermined. From the price index $P = P_{hh}^n(P_{hf})^{1-n}$ and the pricing equations in Table 1, using the labor supply function and its foreign equivalent and the risk-sharing condition, we may have

$$1 = \hat{\lambda\eta}\frac{[E(\frac{C}{\theta})]^n[E(\frac{C}{\theta^*})]^{1-n}}{E(C^{1-\rho})} \tag{B.19}$$

Again, using the log-normality property of solutions, and taking logs, we may solve for the expected (log) consumption for home households

$$Ec = -\frac{1}{\rho}\ln(\hat{\lambda\eta}) - \frac{2-\rho}{2}\sigma_c^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} + \frac{(n\sigma_{cu} + (1-n)\sigma_{cu^*})}{\rho} \tag{B.20}$$

Similarly, we can establishes that

$$Ec^* = -\frac{1}{\rho}\ln(\hat{\lambda\eta}) - \frac{2-\rho}{2}\sigma_{c^*}^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} + \frac{(n\sigma_{c^*u} + (1-n)\sigma_{c^*u^*})}{\rho} \tag{B.21}$$

Solving for EL, EL^*, EU and EU^* Under the LCP specification, the goods market clear condition in the home country is

$$\theta L = n\frac{PC}{P_{hh}} + (1-n)\frac{P^*C^*}{P_{hf}^*} \tag{B.22}$$

Now using the labor supply function, and the pricing equations in Table 1, we can obtain:

$$EL = n\frac{EC^{1-\rho}}{\hat{\lambda\eta}} + (1-n)\frac{EC^{*1-\rho}}{\hat{\lambda\eta}} \tag{B.23}$$

The expected employment of the home country is identical to that of the foreign country. Thus, we may derive the expected utility for the home and foreign households, respectively.

$$EU = EC^{1-\rho}[\frac{\lambda - n(1-\rho)(\lambda-1)}{\lambda(1-\rho)}] - \frac{(1-n)(\lambda-1)}{\lambda}EC^{*1-\rho} \tag{B.24}$$

$$EU^* = EC^{*1-\rho}[\frac{\lambda - (1-n)(1-\rho)(\lambda-1)}{\lambda(1-\rho)}] - \frac{n(\lambda-1)}{\lambda}EC^{1-\rho} \tag{B.25}$$

The expected utility is a combination of two separate functions of the consumption variance and the covariance between the consumption and the productivity shocks.

$$EC^{1-\rho} = \Upsilon \exp(1-\rho)[-\frac{1}{2}\sigma_c^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} + \frac{(n\sigma_{cu} + (1-n)\sigma_{cu^*})}{\rho} \tag{B.26}$$

$$EC^{*1-\rho} = \Upsilon \exp(1-\rho)[-\frac{1}{2}\sigma_{c^*}^2 - \frac{(n\sigma_u^2 + (1-n)\sigma_{u^*}^2)}{2\rho} + \frac{(n\sigma_{c^*u} + (1-n)\sigma_{c^*u^*})}{\rho} \tag{B.27}$$

where $\Upsilon = (\lambda\eta)^{\frac{\rho-1}{\rho}}$.

Solving for monetary policy parameters Under LCP, in log terms, we may write the consumption for the home and foreign country as

$$c - Ec = \frac{m}{\rho} \tag{B.28}$$

$$c^* - Ec^* = \frac{m^*}{\rho} \tag{B.29}$$

Under the unrestricted monetary policy rules, from B.28 and B.29, we can derive the following variance and covariance terms.

$$\sigma_c^2 = \frac{a_1^2\sigma_u^2 + a_2^2\sigma_{u^*}^2}{\rho^2}, \qquad \sigma_{c^*}^2 = \frac{b_1^2\sigma_u^2 + b_2^2\sigma_{u^*}^2}{\rho^2}, \tag{B.30}$$

$$\sigma_{cu} = \frac{a_1\sigma_u^2}{\rho}, \qquad \sigma_{cu^*} = \frac{a_2\sigma_{u^*}^2}{\rho} \tag{B.31}$$

$$\sigma_{c^*u} = \frac{b_1\sigma_u^2}{\rho}, \qquad \sigma_{c^*u^*} = \frac{b_2\sigma_{u^*}^2}{\rho} \tag{B.32}$$

Under the LCP specification, foreign consumption is independent of the parameters of the home country monetary rules. Thus, the home country monetary authority's problem is equivalent to simply minimizing the term $EC^{1-\rho}$. Similarly, the foreign country monetary authority's problem is equivalent to minimizing the term $EC^{*1-\rho}$. The optimal monetary rules in the LCP case which satisfy the international Nash monetary game are

$$a = [n, 1-n], \qquad b = [n, 1-n] \tag{B.33}$$

Checking the currency decision Substituting the policy parameters back into A.23 and A.24, we have

$$\Omega = \Omega^* = 0 \tag{B.34}$$

Weakly, all the firms following LCP is also an equilibrium and the optimal monetary policy associated with this equilibrium results in the fixed exchange rate($s - E(s) = 0$) as the home and foreign monetary authorities respond identically to the home and foreign technology shocks. The expected utility associated with this equilibrium is

$$EU = EU^* = \Theta \exp\{(1-\rho)\frac{(n^2 - \rho n)\sigma_u^2 + ((1-n)^2 - \rho(1-n))\sigma_{u^*}^2}{2\rho^2}\} \tag{B.35}$$

It is straightforward that

$$EU^{PCP} = EU^{*PCP} > EU^{LCP} = EU^{*LCP} \tag{B.36}$$

B.1.3 Case 3 {PCP, LCP}

Now, we analyze the asymmetric case where the firms in one country follow PCP and the firms in the other country follow LCP. Since home and foreign country are complete symmetric in structure, preference and external shocks, we will just focus on the case where home firms follow PCP and foreign firms follow LCP. Note that, in the asymmetric specification, the risk-sharing parameter $\Gamma = \frac{EC^{1-\rho}}{EC^{*1-\rho}}$ is not unity, but both monetary authorities and firms take it as given. The value of Γ has no impact on the choice of optimal monetary rules, but affects the welfare comparison.

Solving for Ec and Ec^* Since all foreign firms choose LCP, the price for home goods and foreign goods in the home country are both predetermined. From the price index of the home country and pricing equations in the Table 1, we have

$$P_{hh}^{n}P_{fh}^{1-n} = \hat{\lambda}\frac{[E(\frac{WC^{1-\rho}}{\theta})]^{n}[E(\frac{W^{*}SC^{1-\rho}}{\theta^{*}})]^{1-n}}{E(C^{1-\rho})} \tag{B.37}$$

Using the risk-sharing condition and the home and foreign labor supply function, taking out the predetermined terms P_{hh} and P_{fh}, we have:

$$1 = \hat{\lambda}\eta\Gamma^{1-n}\frac{[E(\frac{C}{\theta})]^{n}[E(\frac{C}{\theta^{*}})]^{1-n}}{E(C^{1-\rho})}$$

Now using the properties of the log-normal distribution, and taking logs, we may get the expected (log) consumption:

$$Ec = -\frac{1}{\rho}\ln[\hat{\lambda}\eta\Gamma^{1-n}] - \frac{2-\rho}{2}\sigma_{c}^{2} - \frac{n\sigma_{u}^{2}+(1-n)\sigma_{u^*}^{2}}{2\rho} + \frac{n\sigma_{cu}+(1-n)\sigma_{cu^*}}{\rho}$$

Since all home firms choose PCP, the price index of the foreign country, P^* is given by

$$P^{*} = [\frac{P_{hf}}{S}]^{n}P_{ff}^{*\,1-n} \tag{B.38}$$

From B.38, analogously we can obtain:

$$1 = \hat{\lambda}\eta\Gamma^{-n}\frac{[E(\frac{S^{1-n}C^{*}}{\theta})]^{n}[E(\frac{S^{-n}{*}C^{*}}{\theta^{*}})]^{1-n}}{E(C^{*\,1-\rho})} \tag{B.39}$$

This gives

$$\begin{aligned} E(c^{*}) \quad = \quad & -\frac{1}{\rho}\ln(\Gamma^{-n}\hat{\lambda}\eta) - \frac{2-\rho}{2}\sigma_{c^*}^{2} - \frac{n(1-n)}{2\rho}\sigma_{s}^{2} - \frac{n\sigma_{u}^{2}+(1-n)\sigma_{u^*}^{2}}{2\rho} \\ & + \frac{n\sigma_{c^*u}+(1-n)\sigma_{c^*u^*}}{\rho} + \frac{n(1-n)(\sigma_{su}-\sigma_{su^*})}{\rho} \end{aligned} \tag{B.40}$$

Solving for EL, EL^*, EU and EU^* The goods market clearing condition in the home country is

$$\theta L = n\frac{PC}{P_{hh}} + (1-n)\frac{P^{*}C^{*}}{\frac{P_{hf}}{S}} \tag{B.41}$$

Substituting the pricing equations for P_{hh} and P_{hf} into (B.41), we get

$$L = n\frac{PC}{\theta}\frac{E(C^{1-\rho})}{\hat{\lambda}E(\frac{WC^{1-\rho}}{\theta})} + (1-n)\frac{SP^{*}C^{*}}{\theta}\frac{E(C^{*\,1-\rho})}{\hat{\lambda}E(\frac{WC^{*\,1-\rho}}{\theta})} \tag{B.42}$$

Using the labor supply function and the risk-sharing condition, and taking expectation, we can get the expected employment of the home country:

$$EL = \frac{n}{\hat{\lambda}\eta} EC^{1-\rho} + \frac{1-n}{\hat{\lambda}\eta} EC^{*(1-\rho)}\Gamma \tag{B.43}$$

Analogously, we can get:

$$EL^{*} = \Gamma^{-1}\frac{n}{\hat{\lambda}\eta} E(C^{1-\rho}) + \frac{1-n}{\hat{\lambda}\eta} E(C^{*1-\rho}) \tag{B.44}$$

Thus, we can get the expected utility of the home country and foreign country:

$$E(U) = \frac{\lambda - n(\lambda-1)(1-\rho)}{(1-\rho)\lambda} E(C^{1-\rho}) - \frac{(1-n)(\lambda-1)}{\lambda}\Gamma E(C^{*1-\rho}) \tag{B.45}$$

$$EU^{*} = \frac{\lambda - (1-n)(\lambda-1)(1-\rho)}{(1-\rho)\lambda} E(C^{*1-\rho}) - \frac{n(\lambda-1)}{\lambda}\Gamma^{-1} E(C^{1-\rho}) \tag{B.46}$$

Solving for monetary policy parameters Under this asymmetric pricing specification, we may rewrite the exchange rate and the consumption in log terms:

$$s - Es = m - m^{*} \tag{B.47}$$

$$c - Ec = \frac{m}{\rho} \qquad c^{*} - Ec^{*} = \frac{1}{\rho}[nm + (1-n)m^{*}] \tag{B.48}$$

Thus, we may solve for the variances and covariances terms.

$$\sigma_s^2 = (a_1 - b_1)^2\sigma_u^2 + (a_2 - b_2)^2\sigma_{u^*}^2 \tag{B.49}$$

$$\sigma_c^2 = \frac{1}{\rho^2}[a_1^2\sigma_u^2 + a_2^2\sigma_{u^*}^2] \tag{B.50}$$

$$\sigma_{c^*}^2 = \frac{1}{\rho^2}[(na_1 + (1-n)b_1)^2\sigma_u^2 + (na_2 + (1-n)b_2)^2\sigma_{u^*}^2] \tag{B.51}$$

$$\sigma_{cu}^2 = \frac{1}{\rho}a_1\sigma_u^2, \qquad \sigma_{cu^*}^2 = \frac{1}{\rho}a_2\sigma_{u^*}^2 \tag{B.52}$$

$$\sigma_{c^*u}^2 = \frac{1}{\rho}[na_1 + (1-n)b_1]\sigma_u^2, \qquad \sigma_{c^*u^*}^2 = \frac{1}{\rho}[na_2 + (1-n)b_2]\sigma_{u^*}^2 \tag{B.53}$$

$$\sigma_{su} = (a_1 - b_1)\sigma_u^2, \qquad \sigma_{su^*} = (a_2 - b_2)\sigma_{u^*}^2 \tag{B.54}$$

Table 2: The optimal monetary rules under {PCP,LCP}

Parameters	$\rho > 1$	$\rho = 1$
a_1	$\frac{[\rho n+(1-n)]\delta_1 - n(\rho-1)\delta_2}{[\rho n+(1-n)]\delta - n(\rho-1)\delta_2}$	n
a_2	$\frac{[\rho n+(1-n)]\delta_3}{[\rho n+(1-n)]\delta - n(\rho-1)\delta_2}$	$1-n$
b_1	$\frac{-n(\rho-1)\delta_3}{[\rho n+(1-n)]\delta - n(\rho-1)\delta_2}$	0
b_2	$\frac{[\rho n+(1-n)]\delta - n(\rho-1)\delta_2 + n(\rho-1)\delta_3}{[\rho n+(1-n)]\delta - n(\rho-1)\delta_2}$	1
Where	$\delta = \hat{\lambda} - n(1-\rho)\{1+(1-n)[\rho(1-n)+n]\}$	
	$\delta_1 = n\left\{\hat{\lambda} - (1-\rho)[n+(1-n)[\rho(1-n)+n]]\right\}$	
	$\delta_2 = n[(1-n)(1-\rho)]^2$	
	$\delta_3 = (1-n)[\hat{\lambda} - n(1-\rho)]$	
and	$\delta_1 + \delta_3 = \delta$	

Using the same approach, we can derive the optimal monetary rules in Table 2 by solving the international monetary game [6]. The optimal monetary policy rules in Table 2 have the following properties:

$$n \leq a_1 < 1, \quad 0 < a_2 \leq (1-n), \quad b_1 \leq 0, \quad b_2 \geq 1 \tag{B.55}$$

$$a_1 + a_2 = 1, \qquad b_1 + b_2 = 1 \tag{B.56}$$

Checking the currency decision Using these properties of policy parameters, we may rewrite A.23 and A.24 and show that

$$\Omega = \frac{1}{2}(a_1 - b_1)(2 - a_1 - b_1)(\sigma_u^2 + \sigma_{u^*}^2) > 0 \tag{B.57}$$

$$\Omega^* = \frac{1}{2}(a_1 - b_1)(a_1 + b_1)(\sigma_u^2 + \sigma_{u^*}^2) > 0 \tag{B.58}$$

This is because $a_1 - b_1 > 0$ and $2 - a_1 - b_1 > 0$ and $a_1 + b_1 > 0$. These conditions imply all the firms will follow PCP when monetary authorities adopt the optimal monetary rules in Table 2. So the asymmetric pricing specification is not an equilibrium.

[6] See Devereux, Shi and Xu (2003) for more detail.

B.2 Monetary rules chosen before the pricing decision is made

Now we focus on the game where each monetary authority chooses its policy before the currency of pricing decision is made. In general, the maximization problem is quite complex, because the monetary authorities' choice of the monetary rule will partly determine the pricing policies of firms in both countries, and hence will lead to a switching across pass-through outcomes that makes the decision discontinuous. But we can circumvent these difficulties by defining a simpler game, in which the choice set of the monetary authorities in each country is simply binary. Although this seems excessively restrictive, in fact it is not, since we show that this game supports the constrained Pareto optimum of the world economy, which is the flexible price allocation. The game that we focus on allows the monetary authority of each country to choose either the Nash equilibrium monetary rules of the ex-post {PCP, PCP} game defined above, or the Nash equilibrium rules of the ex-post {LCP, LCP} game defined above. The game is then defined in the matrix in Figure 1.

Figure 1: Binary Game

		Foreign Monetary Authority	
		$b = [0,1]$	$b = [n, 1-n]$
Home Monetary Authority	$a = [1,0]$	{PCP, PCP}	{PCP, PCP}
	$a = [n, 1-n]$	{PCP, PCP}	{LCP, LCP}

From this binary game, it can be shown that the {PCP, PCP} configuration with the monetary rules (a=[1,0], b=[0,1]) and flexible exchange rates represent the unique equilibrium of the monetary policy game when monetary authorities take account of the currency of pricing. *Proof*: The proof will be straightforward if we can show the following payoff inequalities

$$EU^{PCP}_{[1,0],[0,1]} > EU^{LCP}_{[n,1-n],[n,1-n]} \tag{B.59}$$

$$EU^{PCP}_{[1,0],[n,1-n]} > EU^{LCP}_{[n,1-n],[n,1-n]} \tag{B.60}$$

B.59 follows directly from B.36. To show B.60, suppose that the foreign monetary authority follows $b = [n, 1-n]$, then if the home country chooses $a = [1, 0]$ and given the {a=([1,0], b=[n,1-n]} configuration, we have

$$\Omega > 0, \qquad \Omega^* > 0 \tag{B.61}$$

So, all the firms will follow PCP. So under this situation, the expected utility for home and foreign country are given by

$$EU^{pcp}_{[1,0],[n,1-n]} = \Theta \exp(1-\rho)\frac{A_1\sigma_u^2 + A_2\sigma_{u^*}^2}{2\rho^2} \tag{B.62}$$

where $A_1 = -n^2(1-n)^2 - n(1-n)^3\rho - n\rho + n^2 + 2n\rho(1-n)^2$ and $A_2 = -(1-n)^4 - n(1-n)^3\rho - (1-n)\rho + 2(1-n)^3 + 2n(1-n)^2\rho$.

Nevertheless, if the home monetary authority choose $a = [n, 1-n]$ instead of $a = [1, 0]$, then the home and foreign monetary rules are the optimal rules associated with the equilibrium of {LCP, LCP}, so the expected utility for this case is

$$EU^{Lcp}_{[n,1-n],[n,1-n]} = \Theta \exp\{(1-\rho)\frac{(n^2-\rho n)\sigma_u^2 + ((1-n)^2 - (1-n)\rho)\sigma_{u^*}^2}{2\rho^2}\} \tag{B.63}$$

We can show

$$A_1 - (n^2 - n\rho) = n(1-n)^2(\rho + \rho n - n) > 0 \tag{B.64}$$

$$A_2 - [(1-n)^2 - (1-n)\rho] = n(1-n)^2(\rho + \rho n - n) > 0 \tag{B.65}$$

Therefore, B.60 holds.

Thus, say that foreign is following $b = [n, 1-n]$. Then if home follows $a = [1, 0]$, it generates a switching to all PCP, and then $a = [1, 0]$ is clearly better for home since B.60 holds. Similarly, if foreign is following $b = [0, 1]$, then the optimal policy for home is to follow $a = [1, 0]$. So the {PCP, PCP} configuration with the monetary rules (a=[1,0], b=[0,1]) and flexible exchange rates represent the unique equilibrium of the ex-ante monetary policy game, which also supports the full flexible price equilibrium.

B.3 Equilibrium under restricted monetary policy rules

We now use the same approach to find the equilibrium under restricted monetary policy rules. The restriction of the monetary policy rules will affect the level of the expected utility by changing the variance and covariance terms.

B.3.1 Case 1 {PCP, PCP}

First, we may derive the exchange rate and consumption in log terms:

$$s - E(s) = m - m^* \tag{B.66}$$

$$c - Ec = \frac{nm + (1-n)m^*}{\rho} \tag{B.67}$$

Under the restricted monetary policy rules, from B.66 and B.67, we can derive the following variance and covariance terms.

$$\sigma_s^2 = a^2\sigma_u^2 + b^2\sigma_{u^*}^2, \qquad \sigma_c^2 = \frac{n^2a^2\sigma_u^2 + (1-n)^2b^2\sigma_{u^*}^2}{\rho^2}, \tag{B.68}$$

$$\sigma_{cu} = \frac{na\sigma_u^2}{\rho}, \qquad \sigma_{cu^*} = \frac{(1-n)b\sigma_{u^*}^2}{\rho} \tag{B.69}$$

$$\sigma_{su} = a\sigma_u^2, \qquad \sigma_{su^*} = -b\sigma_{u^*}^2 \tag{B.70}$$

Substituting the above terms into B.8, and solving the international monetary game, we have

$$a = 1, \qquad b = 1 \tag{B.71}$$

Then using A.25 and A.26, we find

$$\Omega = \Omega^* = \frac{1}{2}(\sigma_u^2 + \sigma_{u^*}^2) > 0 \tag{B.72}$$

Thus, all firms choosing PCP is an equilibrium and the optimal monetary policy associated with this equilibrium ensures the flexible exchange rate where $s - E(s) = u - u^*$. The expected utility associated with this equilibrium is exactly the same as in B.18.

B.3.2 Case 2 {LCP, LCP}

Under LCP, we may get the home and foreign countries log consumption:

$$c - Ec = \frac{m}{\rho} \qquad c^* - Ec^* = \frac{m^*}{\rho} \tag{B.73}$$

Under the restricted monetary policy rule, from B.73, we can derive the following variance and covariance terms.

$$\sigma_c^2 = \frac{a^2\sigma_u^2}{\rho^2}, \qquad \sigma_{c^*}^2 = \frac{b^2\sigma_{u^*}^2}{\rho^2}, \tag{B.74}$$

$$\sigma_{cu} = \frac{a\sigma_u^2}{\rho}, \qquad \sigma_{cu^*} = 0 \tag{B.75}$$

$$\sigma_{c^*u} = 0, \qquad \sigma_{c^*u^*} = \frac{b\sigma_{u^*}^2}{\rho} \tag{B.76}$$

Using a similar approach as in above sections, we can derive

$$a = n, \qquad b = 1 - n \tag{B.77}$$

This gives us

$$\Omega = \frac{n(2-n)}{2}\sigma_u^2 + \frac{(1-n)^2}{2}\sigma_{u^*}^2 > 0, \qquad \Omega^* = \frac{n^2}{2}\sigma_u^2 + \frac{1-n^2}{2}\sigma_{u^*}^2 > 0 \tag{B.78}$$

This implies, all the firms will follow PCP. Thus {LCP, LCP} is not an equilibrium when monetary authorities can only use the restricted money rules.

B.3.3 Case 3 {PCP, LCP}

Under this asymmetric pricing specification, the exchange rate and consumption in log terms are:

$$s - Es = m - m^* \tag{B.79}$$

$$c - Ec = \frac{m}{\rho} \qquad c^* - Ec^* = \frac{1}{\rho}[nm + (1-n)m^*] \tag{B.80}$$

Under restricted monetary policy rules, we may solve for the variances and covariances terms.

$$\sigma_s^2 = a^2\sigma_u^2 + b^2\sigma_{u^*}^2 \tag{B.81}$$

$$\sigma_c^2 = \frac{a^2\sigma_u^2}{\rho^2} \tag{B.82}$$

$$\sigma_{c^*}^2 = \frac{1}{\rho^2}[n^2a^2\sigma_u^2 + (1-n)^2b^2\sigma_{u^*}^2] \tag{B.83}$$

$$\sigma_{cu}^2 = \frac{1}{\rho}a\sigma_u^2, \qquad \sigma_{cu^*}^2 = 0 \tag{B.84}$$

$$\sigma_{c^*u}^2 = \frac{na}{\rho}\sigma_u^2, \qquad \sigma_{c^*u^*}^2 = \frac{(1-n)b}{\rho}\sigma_{u^*}^2 \tag{B.85}$$

$$\sigma_{su} = a\sigma_u^2, \qquad \sigma_{su^*} = -b\sigma_{u^*}^2 \tag{B.86}$$

Using the same methodology, we can derive the following solution

$$a = n\frac{\hat{\lambda} - (1-\rho)[n + (1-n)(\rho - \rho n + n)]}{\hat{\lambda} - n(1-\rho)[1 + (1-n)(\rho - \rho n + n)]}, \qquad b = 1 \tag{B.87}$$

It can be shown that $n \leq a < 1$ [7]. It gives us

$$\Omega = \frac{a(2-a)}{2}\sigma_u^2 + \frac{1}{2}\sigma_{u^*}^2 > 0, \qquad \Omega^* = \frac{a^2}{2}\sigma_u^2 + \frac{1}{2}\sigma_{u^*}^2 > 0 \tag{B.88}$$

In each country, firms would wish to set prices in their own currency. Hence, the {PCP, LCP} configuration is *not* an equilibrium if restricted monetary policy rules are chosen by the monetary authorities, taking the currency of pricing as given.

References

[1] Devereux, Michael B., and Charles Engel (2003). "Monetary Policy in the Open Economy Revisited: Price setting and Exchange rate flexibility," *Review of Economic Studies*, 70, 765-84.

[2] Devereux, Michael B., Kang Shi, and Juanyi Xu (2003). "Global Monetary Policy under a Dollar Standard," *CEPR Working Paper*, No. 4317.

[7] In the special case with $\rho = 1$, we have $a = n$.